

Spectral Analysis of Gluonic Pole Matrix Elements

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We use a spectator framework to investigate the spectral properties of quark-quark-gluon correlators and use this to study gluonic pole matrix elements. Such matrix elements appear in principle both for distribution functions such as the Sivers function and fragmentation functions such as the Collins function. We find that the contribution of the gluonic pole matrix element in fragmentation functions vanishes. This outcome is important in the study of universality for fragmentation functions.

We investigate multi-parton correlators with one additional gluon in which the zero-momentum limit will be studied [1, 2]. These are so-called gluonic pole matrix elements or Qiu-Sterman matrix elements, that have opposite time-reversal (T) behavior as compared to the matrix elements without the gluon. Such matrix elements involving time-reversal odd (T-odd) operator combinations are of interest because they are essential for understanding single spin asymmetries in high energy scattering processes e.g. semi-inclusive deep inelastic scattering (SIDIS) and Drell-Yan scattering. In order to understand the basic features of these matrix elements we perform a spectral analysis by modeling the distribution and fragmentation functions under *reasonable* assumptions [3]. In particular we consider the differences between distribution and fragmentation functions using a spectral analysis while restricting the momentum dependence and asymptotic behavior of the vertices. In this context, the relevant gluonic pole matrix elements that we want to study are $\Phi_G(k, k - k_1)$ and $\Delta_G(k, k - k_1)$ shown in Figs. 1 and 2. Of these matrix elements only the dependence on the collinear components x and x_1 in the expansion of the momenta is needed (note, the gluon momentum is parameterized as $k_1 = [k_1^-, x_1, k_{1T}]$ in these figures). We find that while both $\Phi_G(x, x - x_1)$ and $\Delta_G(x, x - x_1)$ are nonzero, taking the limit $x_1 \rightarrow x$, $\Phi_G(x, x)$ remains non-zero, while $\Delta_G(x, x)$ vanishes. The vanishing of the T-odd gluonic pole matrix elements is important in the study of universality of transverse momentum dependent (TMD) distribution and fragmentation functions (FFs).

T-odd *operator structure* can be traced to the color gauge link that necessarily appears in correlators to render them color gauge-invariant [4, 5]. The quark-quark correlator depending on the collinear and transverse components of the quark momentum, $k = x P + \sigma n + k_T$ (where the Sudakov vector n is an arbitrary light-like four-vector $n^2 = 0$ that has non-zero overlap $P \cdot n$ with the hadron's momentum P and $k^- \sim \sigma$ which is suppressed w/r to the hard scale) is given by,

$$\Phi_{ij}^{[\mathcal{U}]}(x, k_T) = \int \frac{d(\xi \cdot P) d^2 \xi_T}{(2\pi)^3} e^{ik \cdot \xi} \langle P | \bar{\psi}_j(0) \mathcal{U}_{[0;\xi]} \psi_i(\xi) | P \rangle \Big|_{\text{LF}} , \quad (1)$$

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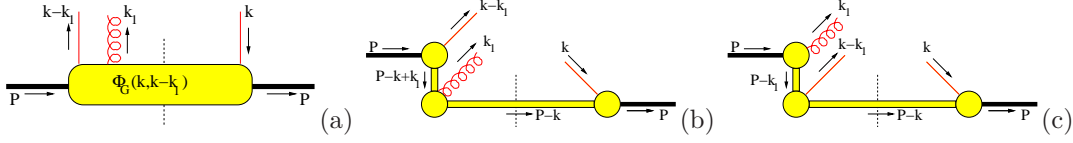


Figure 1: The graphical representation of the quark-quark-gluon correlator Φ_G for the case of distributions including a gluon with momentum k_1 (a), and the possible intermediate states (b) and (c) in a spectator model description. Conjugate contribution to (b) and (c) not shown.

where LF ($\xi \cdot n = 0$) designates the light-front. The *gauge link* is the path-ordered exponential, $\mathcal{U}_{[\eta;\xi]} = \mathcal{P} \exp \left[-ig \int_C ds \cdot A^a(s) t^a \right]$ along the integration path C with endpoints at η and ξ . Its presence in the hadronic matrix element is required by gauge-invariance. Similarly, the fragmentation correlator depending on the collinear and transverse components of the quark momentum, $k = \frac{1}{z} P + k_T + \sigma n$, is given by [5]

$$\Delta_{ij}^{[\mathcal{U}]}(z, k_T) = \sum_X \int \frac{d(\xi \cdot P_h) d^2 \xi_T}{(2\pi)^3} e^{i k \cdot \xi} \langle 0 | \mathcal{U}_{[0,\xi]} \psi_i(\xi) | P, X \rangle \langle P, X | \bar{\psi}_j(0) | 0 \rangle_{LF}. \quad (2)$$

In the correlators the integration path C in the gauge link designates process-dependence. This is due to the observation that the operator structure of the correlator is also a consequence of the necessary resummation of all contributions that arise from collinear gluon polarizations, i.e. those along the hadron momentum. How this resummation takes effect is a matter of calculation [6]. The result is a process dependence in the path in the gauge link. After azimuthal weighting of cross sections one simply finds that the T-odd features originating from the gauge link lead to specific factors with which the T-odd functions appear in observables. For example, comparing T-odd effects in DFs in SIDIS and the Drell-Yan process one finds a relative minus sign [7, 8]. Similarly, comparing T-odd effects in FFs in SIDIS and electron-positron annihilation one also finds a relative minus sign for the T-odd effect originating from the operator structure (gauge link) [5]. The T-odd operator parts are precisely the soft limits ($k_1 \rightarrow 0$ or x_1 and $z^{-1} = x_1 \rightarrow 0$) of the gluonic pole matrix elements [5]

$$\Phi_{\partial}^{\alpha [\mathcal{U}]}(x) = \tilde{\Phi}_{\partial}^{\alpha}(x) + C_G^{[\mathcal{U}]} \pi \Phi_G^{\alpha}(x, x), \quad \text{and} \quad \Delta_{\partial}^{\alpha [\mathcal{U}]}(z) = \tilde{\Delta}_{\partial}^{\alpha}\left(\frac{1}{z}\right) + C_G^{[\mathcal{U}]} \pi \Delta_G^{\alpha}\left(\frac{1}{z}, \frac{1}{z}\right) \quad (3)$$

(see Figs. 1 and 2). They arise in the decomposition of the transverse weighted quark correlators

$$\Phi_{\partial}^{\alpha [\mathcal{U}]}(x) = \int d^2 k_T k_T^{\alpha} \Phi^{[\mathcal{U}]}(x, k_T), \quad \text{and} \quad \Delta_{\partial}^{\alpha [\mathcal{U}]}(z) = \int d^2 k_T k_T^{\alpha} \Delta^{[\mathcal{U}]}(z, k_T), \quad (4)$$

which are the relevant operators in analyzing the azimuthal asymmetries. The process-dependent gluonic pole factors $C_G^{[\mathcal{U}]}$ are calculable and the process (link) independent correlators $\tilde{\Phi}_{\partial}$ and $\tilde{\Delta}_{\partial}$ contains the T-even operator combination, while Φ_G and Δ_G contain the T-odd operator combination. The latter one is precisely the soft limit, $z_1^{-1} = x_1 \rightarrow 0$,

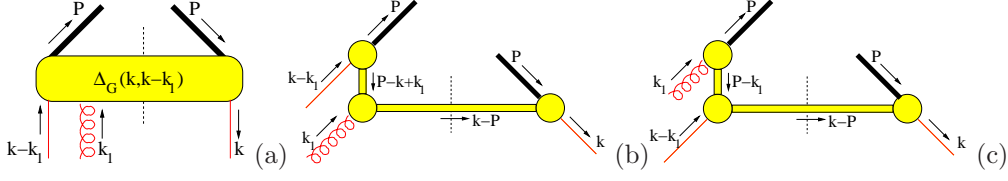


Figure 2: The graphical representation of the quark-quark-gluon correlator Δ_G in the case of fragmentation including a gluon with momentum k_1 (a) and the possible intermediate states (b) in a spectator model description. Conjugate contribution to (b) and (c) not shown

of the quark-gluon correlator $\Delta_{Gij}^\alpha(x, x_1)$

$$\begin{aligned} \Delta_{Gij}^\alpha(x, x-x_1) &= \sum_X \int \frac{d(\xi \cdot P)}{2\pi} \frac{d(\eta \cdot P)}{2\pi} e^{i x_1(\eta \cdot P)} e^{i(x-x_1)(\xi \cdot P)} \\ &\quad \times \langle 0 | \mathcal{U}_{[0,\eta]}^n g G^{n\alpha}(\eta) \mathcal{U}_{[\eta,\xi]}^n \psi_i(\xi) | P, X \rangle \langle P, X | \bar{\psi}_j(0) | 0 \rangle \Big|_{LC}. \end{aligned}$$

Because of the appearance of hadronic states $|P, X\rangle$, each of correlators in $\Delta_\partial^{\alpha[\mathcal{U}]}(z)$ contains in principle T-even and T-odd functions. However, rather than having a doubling of T-odd functions, we find that $\Delta_G(x, x) = 0$, which implies that T-odd fragmentation functions appear in the matrix elements of the T-even operator combination in $\tilde{\Delta}_\partial$ involving a hadron-jet state (non-plane-wave). They are process independent for instance the T-odd Collins function [9] and they appear with a universal strength (no gluonic pole factors). In contrast T-odd DFs in Φ_∂ only can come from $\Phi_G(k_1 = 0)$. These DFs can still be universal but appear with calculable process dependent gluonic pole factors [10, 6].

To see this in a spectator model approach, we consider the distribution or fragmentation correlators with a spectator with mass M_s . The result for the cut, but untruncated, diagrams, such as in Figs. 1 and 2 (without the gluon insertion) are of the form

$$\Phi(x, k_T) \sim \int d(k \cdot P) \frac{F(k^2, k \cdot P)}{(k^2 - m^2 + i\epsilon)^2} \delta((k - P)^2 - M_s^2),$$

where $F(k^2, k \cdot P)$ contains the numerators of propagators and/or traces of them in the presence of Dirac Gamma matrices, as well as the vertex form factors (see for example [11]). In the above the delta function constraint in Eq. 5 has been implemented. One finds that the numerator $F(k^2, k \cdot P) = F(x, k_T^2)$ and hence

$$\Phi(x, k_T) \sim \frac{(1-x)^2 F(x, k_T)}{(\mu^2(x) - k_T^2)^2}, \quad (5)$$

with $\mu^2(x) = x M_s^2 + (1-x) m^2 - x(1-x) M^2$. Note that $k_T^2 = -\mathbf{k}_T^2 \leq 0$. The details of the numerator function depend on the details of the model, including the vertices, polarization sums, etc. These must be chosen in such a way as to not produce unphysical effects, such as a decaying proton if $M \geq m + M_s$, thus m in Eq. 5 must represent some constituent mass in the quark propagator, rather than the bare mass. The useful feature of the result in Eq. 5 is its ability to produce reasonable valence and even sea quark distributions using the freedom

in the model. The results for the fragmentation function in the spectator model is identical upon the substitution of $x = 1/z$ [11].

We now turn to the same spectral analysis of the gluonic pole correlator using the picture given in Figs. 1 and 2 for distribution and fragmentation functions respectively. Again, we only need to investigate one of the cases. Parameterizing the gluon momentum as $k_1 = [k_1^-, x_1, k_{1T}]$, $k_1^- = k_1 \cdot P - \frac{1}{2} x_1 M^2$ is the first component to be integrated over [3]. Assuming that the numerator does not grow with k_1^- one can easily perform the k_1^- integrations assuming that the F_i are independent of k_1^- . Taking the limit $x_1 \rightarrow 0$ of the basic result for the quark-gluon correlators $\Phi_G(x, x-x_1, k_T, k_T-k_{1T})$ we obtain the gluonic pole correlators, for distribution functions ($0 \leq x \leq 1$) (see [3] for details)

$$\Phi_G(x, x) = - \int d^2 k_T d^2 k_{1T} \frac{(1-x) F_1(x, 0, k_T, k_{1T}) \theta(1-x)}{(\mu^2 - k_T^2)(x B_1 + (1-x) A_2) A_1}, \quad (6)$$

where $A_i(\{m_i^2\}, \{k_{iT}^2\}, \{x_i\})$, and for fragmentation functions ($x = 1/z \geq 1$)

$$\Delta_G(x, x) = 0. \quad (7)$$

This result depends on the assumption that the numerator does not grow with k_1^- , otherwise, one does not get the required $x_1 \theta(x_1)$ behavior in the calculation [3]. In models, terms proportional to $k_1^- \sim k_1 \cdot P$ may easily arise from numerators of fermionic propagators [12] which may easily be suppressed by form factors at the vertices. To prove a proper behavior within QCD one would need to study the fully unintegrated correlators such as e.g. in Ref. [13] and show that they fall off sufficiently fast as a function of $k_1 \cdot P$.

While our analysis is not yet the full proof that gluonic pole matrix elements vanish in the case of fragmentation, it is a step towards such a proof and the possible direction to obtain such a proof by considering the appropriate color gauge-invariant soft matrix elements. Such a proof is important as it eliminates a whole class of matrix elements parameterized in terms of T-odd fragmentation functions besides the T-odd fragmentation functions in the parameterization of the two-parton correlators.

References

- [1] A. V. Efremov and O. V. Teryaev, *Sov. J. Nucl. Phys.*, 36, 140 (1982).
- [2] Jian-Wei Qiu and George Sterman, *Phys. Rev. Lett.*, 67, 2264 (1991).
- [3] L. P. Gamberg, A. Mukherjee, and P. J. Mulders, arXiv-0803.2632, to be published in *Phys. Rev. D*.
- [4] Andrei V. Belitsky, X. Ji, and F. Yuan, *Nucl. Phys.*, B656, 165 (2003).
- [5] Daniel Boer, P. J. Mulders, and F. Pijlman, *Nucl. Phys.*, B667, 201 (2003).
- [6] C. J. Bomhof and P. J. Mulders, *JHEP*, 02, 029 (2007).
- [7] John C. Collins, *Phys. Lett.*, B536, 43 (2002).
- [8] Stanley J. Brodsky, Dae Sung Hwang, and Ivan Schmidt, *Nucl. Phys.*, B642, 344 (2002).
- [9] John C. Collins, *Nucl. Phys.*, B396, 161 (1993).
- [10] C. J. Bomhof, P. J. Mulders, and F. Pijlman, *Phys. Lett.*, B596, 277 (2004).
- [11] R. Jakob, P. J. Mulders, and J. Rodrigues, *Nucl. Phys.*, A626, 937 (1997).
- [12] L. P. Gamberg, D. S. Hwang, A. Metz, and M. Schlegel, *Phys. Lett.*, B639, 508 (2006).
- [13] J. C. Collins, T. C. Rogers, and A. M. Stasto, *Phys. Rev.*, D77, 085009 (2008).